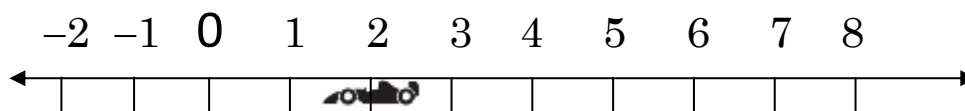


AVERAGE VELOCITY

Now we look at a number line — a simple horizontal axis — and talk about some object moving left or right on the line.



It could be a racecar speeding down the backstretch or a subatomic proton hurtling through a linear accelerator.

□ NOTATION

Throughout this chapter, we'll let t represent *time* and s represent the *position* of the object on the line. We agree that $s > 0$ (positive) when the object is to the right of 0, and $s < 0$ (negative) on the left side of 0. It is also important that we consider the position, s , to be a function of time, t . In other words, as the time changes, the position of the object may also change. We can even write the position as $s(t)$ [read: " s of t "] to express the view that the position, s , is a function of the time, t .

In addition, to emphasize the math, we won't worry about units. But for your edification, if position is measured in meters from the origin (as either a positive number, negative number, or zero) and time is measured in seconds (common in physics), then the

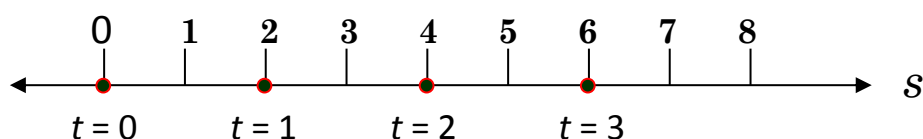
Speed v. Velocity

65 mph is *speed*.
65 mph To The Right is *velocity*.

unit of **velocity** is *meters per second*, or m/s, or $\frac{\text{m}}{\text{s}}$, followed by the direction “left” or “right.”

□ **TIME AND POSITION**

Consider the following real line (which we could call the ***s***-axis). Each dot on the line represents the **position** of an object at the moment of **time** specified, where t might be measured in seconds:



Here’s what this little picture means. At the “beginning,” when $t = 0$, the position is $s = 0$. One second later, when $t = 1$, the object is at $s = 2$. One second after that, when $t = 2$, s has the value 4. When $t = 3$, the object is at 6. Expressing these facts in function notation, we write:

$$s(0) = 0 \qquad s(1) = 2 \qquad s(2) = 4 \qquad s(3) = 6$$

[The notation ***s(t)*** is read “*s of t.*”]

We can also express this number line picture with a table of values:

t	0	1	2	3
s	0	2	4	6

Our goal now is to predict where the object will be at some time in the **future**. For example, when $t = 10$, that is, 10 seconds after the object begins moving, where will it be? To do this, we ask the question: Given the data in the table, can we find a formula for the position of the object as a function of time? Looking either at the real line (the s -axis) or the table, it seems (at least from the data we have) that the position is always twice the value of time: ***s is twice t***. So we can write the formula

$$s(t) = 2t \qquad \text{[The position at time } t \text{ is twice the } t\text{-value]}$$

Thus, to predict the position s when $t = 10$, all we have to do is calculate $s(10)$, which is $2 \cdot 10 = 20$. Therefore, after 10 seconds have passed, the object is at **position 20**. Notice that the object in this example never has a negative value of s , and that it moves to the right as the value of time increases.

Homework

- The following table represents the position of a car at various moments in time. Predict the position of the car when $t = 850$.

t	1	2	3	4	5
s	4	7	12	19	28

□ AVERAGE VELOCITY

Have you studied **delta** notation? A change in y , which might be written as the difference $y_2 - y_1$, can be expressed as Δy . In discussing velocity, we need the notion of a change in time, denoted by Δt , and as a consequence, a change in position, written Δs .

The symbol Δ is the Greek capital letter **delta**, and is a common symbol representing a **change** in something.

Now we're ready to define the concept of **average velocity**:

Average Velocity = the change in position *divided by* the change in time.

It's a measure of the *net distance traveled* during a specified amount of time. We use the symbol $\bar{v}$ (v-bar) to represent average velocity:

$$\bar{v} = \frac{\Delta s}{\Delta t}$$

This formula is very similar to the formula involving distance, rate, and time:

$$r = \frac{d}{t}$$

Average Velocity

Four Notes:

1. The change in position, Δs , does not take into account the actual distance traveled by the object. Suppose you and I are standing on the s -axis at the same spot, say $s = 10$. You run many miles to the left and then turn around and run back, ending at $s = 12$. Your change in position is $12 - 10 = 2$. I, being the lazy bum that I am, simply stroll directly from $s = 10$ to $s = 12$. My change in position is $12 - 10 = 2$, the same change in position as yours. We started at the same spot and we ended at the same spot, and so we each experienced the same change in position — our Δs 's are the same (even though you ran a much farther distance than I did.)
2. The formula for average velocity may remind you of the definition of **slope**, $m = \frac{\Delta y}{\Delta x}$. This is no coincidence. Each of the concepts — *slope* and *average velocity* — is described by a ratio of changes, and each of them forms the foundation for the branch of math called *calculus*.
3. Just as the slope of a line can be negative, the average velocity may be negative. You'll see this in Example 2 below. By the way, this is one of the reasons that we refer to the theme of this chapter using the term *velocity* rather than speed.
4. You might notice at this point that your calculation for average velocity will involve two subtractions followed by a division. The order in which you perform the subtractions is up to you, but you **MUST** be consistent in that order. Nevertheless, tradition dictates that we subtract initial values from final values. For example, if an object is at $s = 3$ and then later is at $s = 10$, we agree to calculate $10 - 3$ rather than $3 - 10$. So our average velocity formula looks like

Velocity
has
direction;
speed
does not.

$$\bar{v} = \frac{\text{final position} - \text{initial position}}{\text{final time} - \text{initial time}}$$

Average Velocity

EXAMPLE 1: A bird takes off from position $s = 3$ when $t = 5$ and lands at position $s = 15$ when $t = 9$. Find the average velocity for the time interval $t = 5$ to 9.

Solution: The average velocity is the change in position (the difference of the positions) divided by the change in time (the difference of the times):

$$\bar{v} = \frac{\Delta s}{\Delta t} = \frac{15-3}{9-5} = \frac{12}{4} = \boxed{3}$$

EXAMPLE 2: A racecar is at position $s = 8$ when $t = 0$ and then drives to position $s = -10$ in 2 seconds. Find the average velocity during the 2-second interval.

Solution: The racecar started when $t = 0$ and ended when $t = 2$. Using the formula for average velocity, we calculate

$$\bar{v} = \frac{\Delta s}{\Delta t} = \frac{-10-8}{2-0} = \frac{-18}{2} = \boxed{-9}$$

Why is the velocity *negative*? Because the racecar moved to the *left*.

Homework

2. A snail is at position $s = 22$ at $t = 4$ and moves to position $s = 50$ at $t = 18$. Find the average velocity, $\bar{v}$, for the time interval $t = 4$ to 18.
3. A rocket is at position $s = 1,000$ at $t = 20$ and rises to position $s = 5,500$ at $t = 65$. Find the average velocity, $\bar{v}$.

EXAMPLE 3: Let $s(t) = t^2$. Calculate each of the following:

- a. $s(0)$ b. $s(20)$ c. $\bar{v}$ from $t = 5$ to $t = 15$

Solution:

- a. $s(0)$ means let $t = 0$ and calculate s using the given formula:

$$s(0) = 0^2 = \mathbf{0}$$

- b. $s(20)$ is the position when $t = 20$:

$$s(20) = 20^2 = \mathbf{400}$$

- c. i) $s(5) = 5^2 = 25$

ii) $s(15) = 15^2 = 225$

t	s
0	0
5	25
15	225
20	400

The average velocity, $\bar{v}$, is calculated as follows:

$$\bar{v} = \frac{\Delta s}{\Delta t} = \frac{225 - 25}{15 - 5} = \frac{200}{10} = \mathbf{20}$$

Homework

For each position function, calculate the average velocity, $\bar{v}$, during the given time interval:

4. $s(t) = 3t + 7$ $t = 3$ to $t = 10$

5. $s(t) = t^2 + 3t + 7$ $t = 1$ to $t = 5$

6. $s(t) = t^3$ $t = 3$ to $t = 5$

7. $s(t) = t^3 + t^2 + 3t + 7$ $t = 1$ to $t = 2$

8. $s(t) = \frac{1}{t}$ $t = 4$ to $t = 6$
9. $s(t) = \sqrt{t}$ $t = 9$ to $t = 36$
10. $s(t) = \frac{1}{\sqrt{t}}$ $t = 4$ to $t = 100$
11. $s(t) = 7$ $t = \pi$ to $t = 50$
12. For each position function, calculate the average velocity, $\bar{v}$, during the time interval from $t = 0$ to $t = 1$:
 - a. $s(t) = at + b$
 - b. $s(t) = at^2 + bt + c$
 - c. $s(t) = at^3 + bt^2 + ct + d$

Solutions

- | | |
|---|-------------------|
| 1. 722,503 | 2. 2 |
| 3. 100 | 4. 3 |
| 5. 9 | 6. 49 |
| 7. 13 | 8. -0.04 |
| 9. 0.11 | 10. -0.004 |
| 11. 0 | |
| 12. I want to see what you come up with. | |

“STRIVE FOR *progress*,
NOT *perfection*.”